# Efficient Generation of Geographically Accurate Transit Maps

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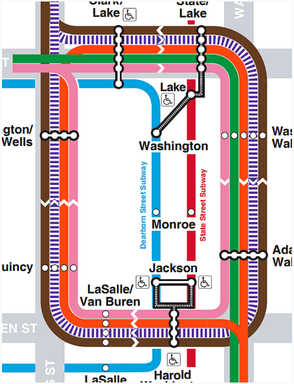
November 7, 2018

# Motivation

Official CTA map



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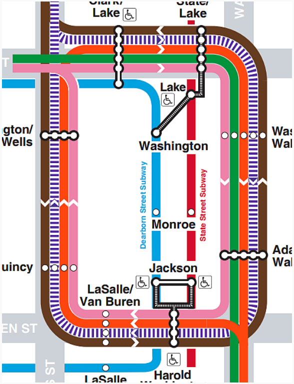
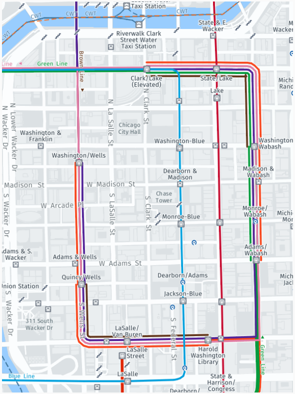



HERE



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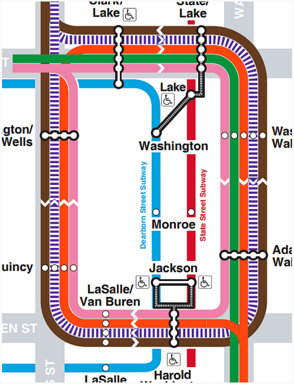


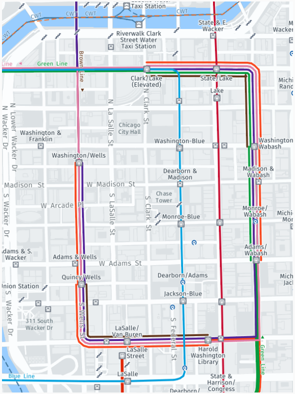


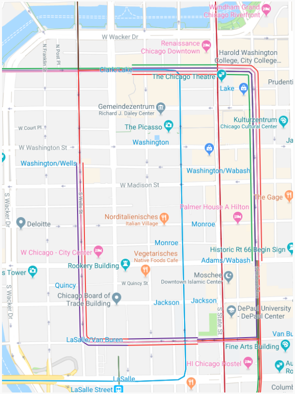

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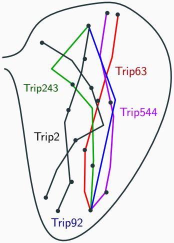


”Bag of trips”
(GTFS)

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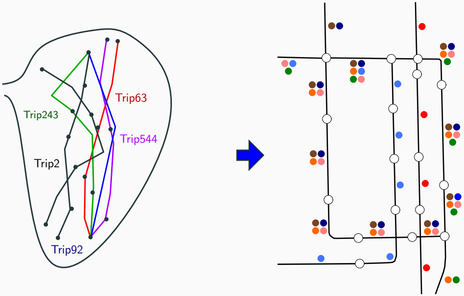


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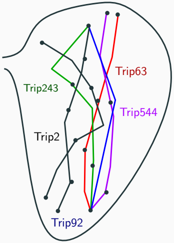


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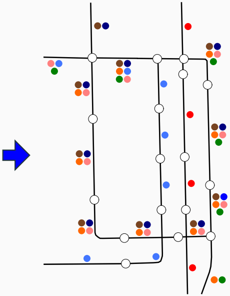

Line graph

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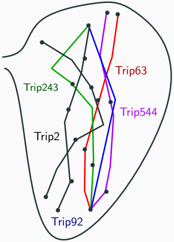


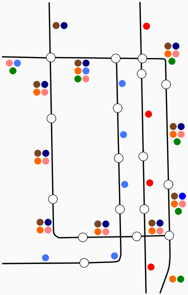

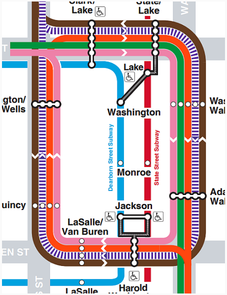


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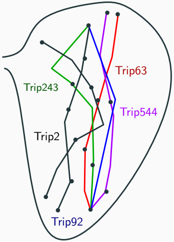


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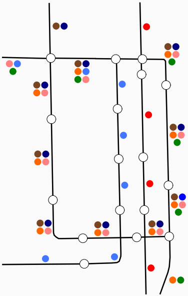

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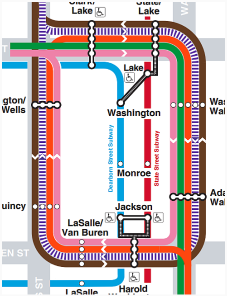
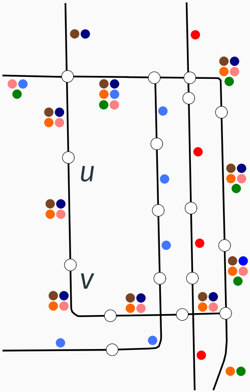



Final map

# Line graph construction

Line graph:

- Undirected labeled graph $G = (V, E, L)$



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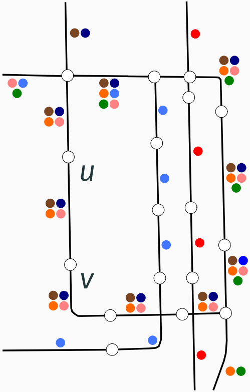

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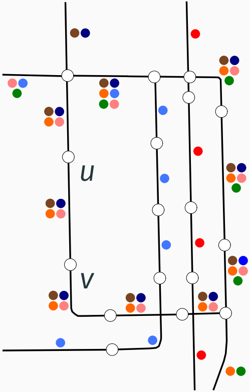

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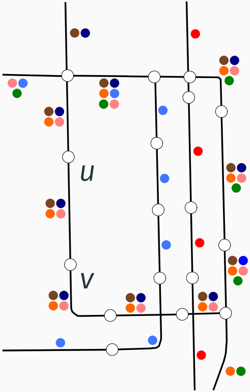
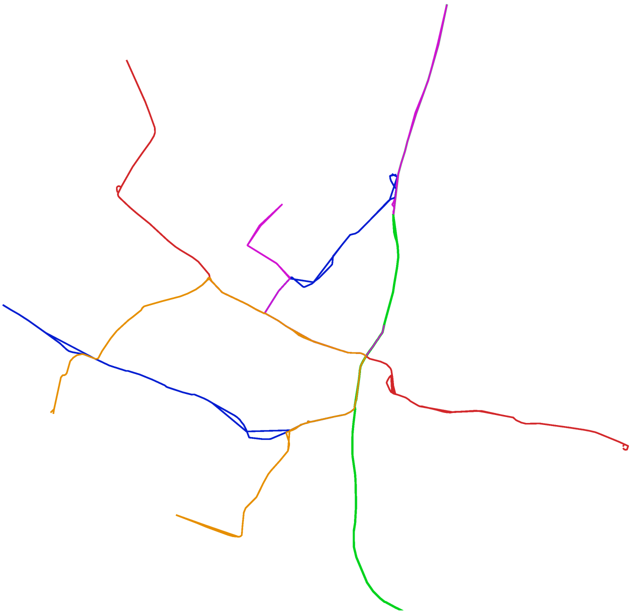



Example: $\mathcal{L} = \{$$\}$, $L((u, v)) = \{$$\}$

# Line graph construction - Input data



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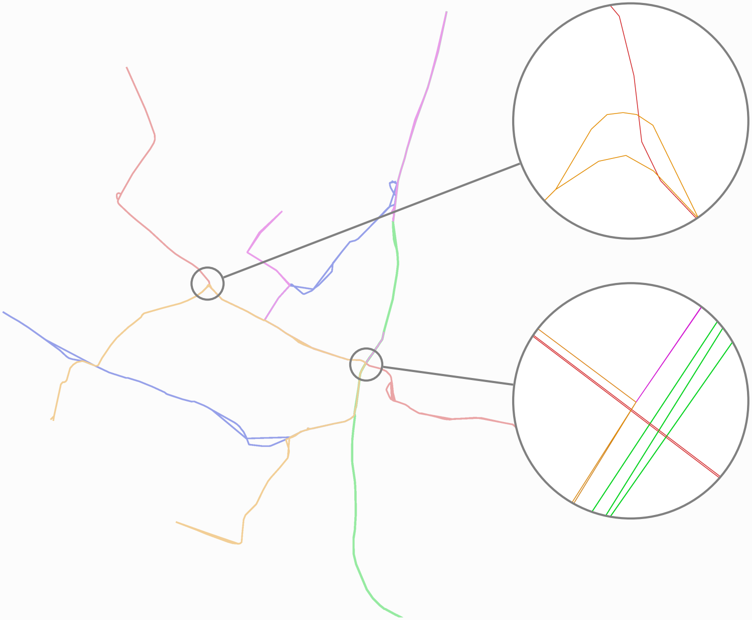

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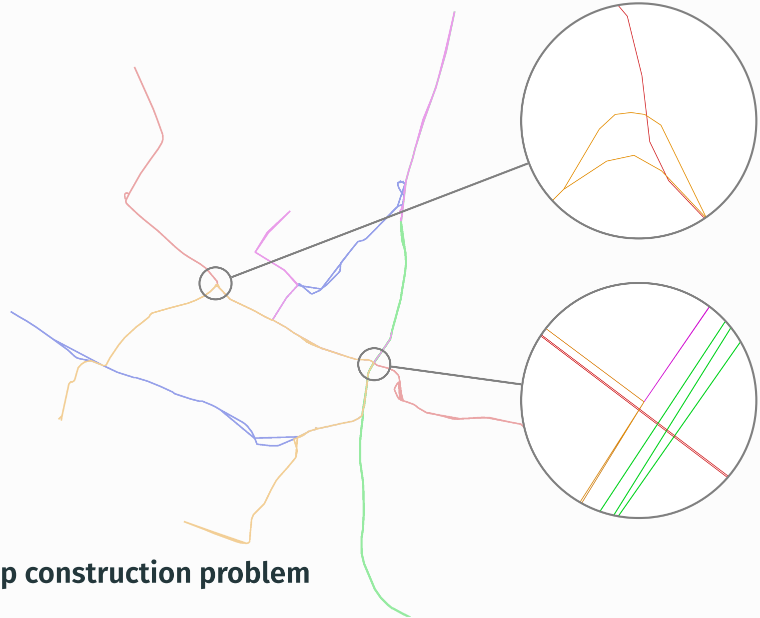
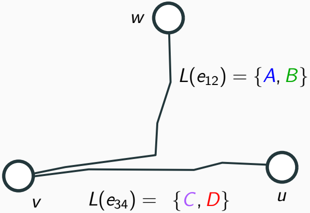


⇒ **Map construction problem**

# Line graph construction - Non-station nodes



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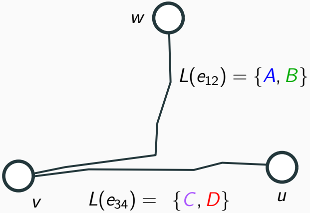


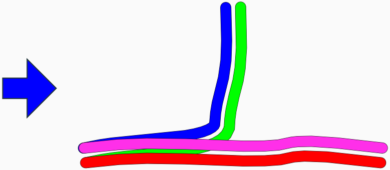

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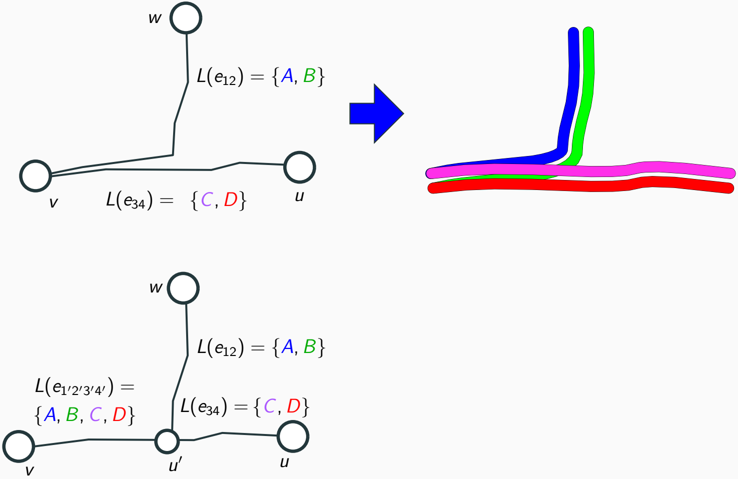

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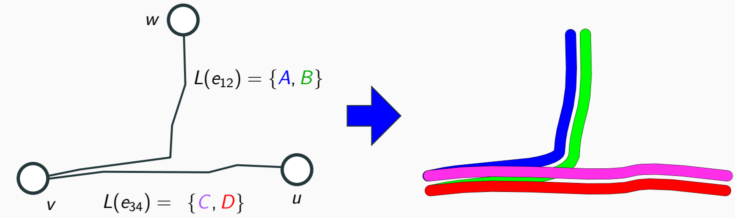


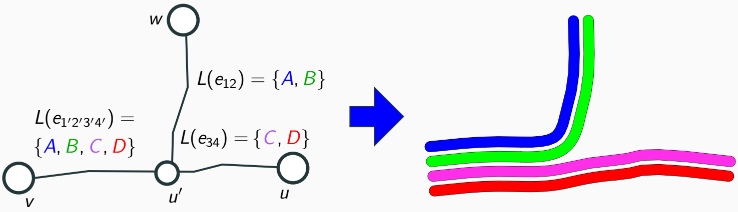

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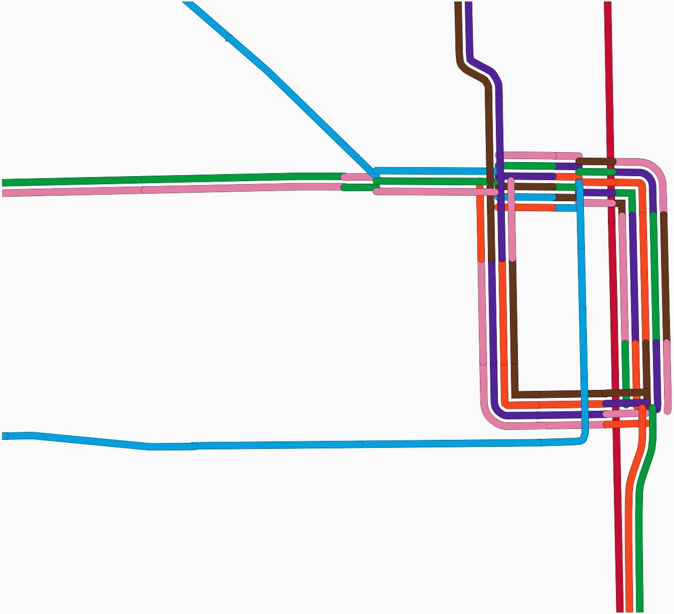

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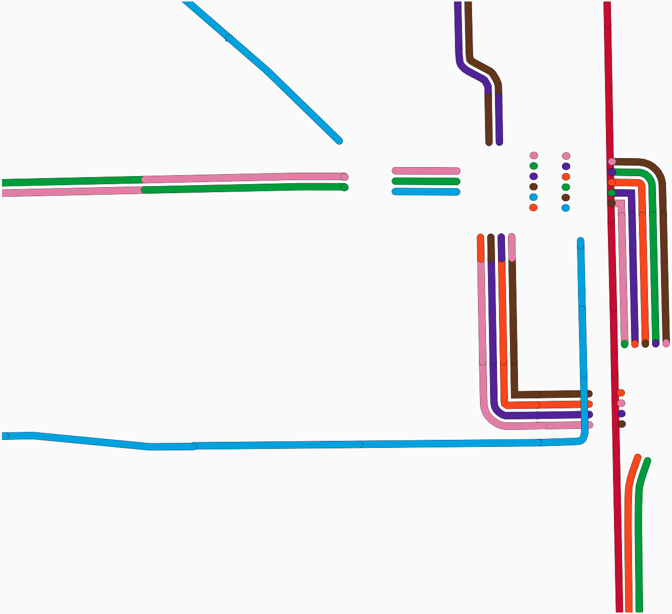

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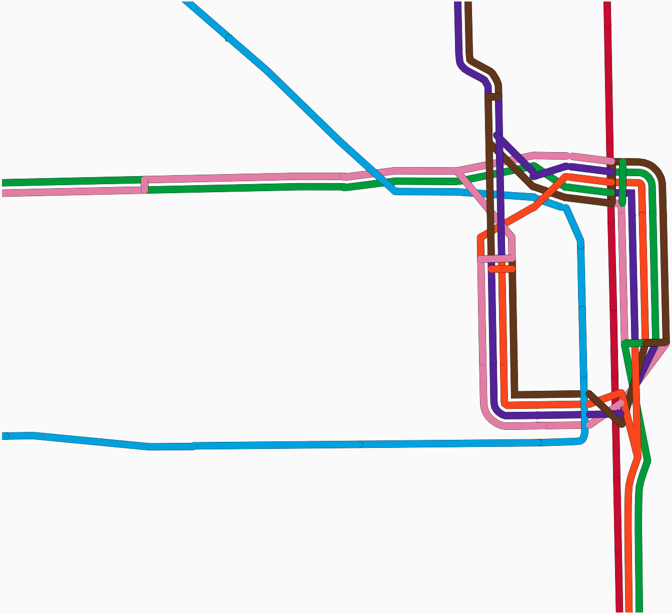
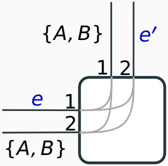


# Line-ordering optimization - Baseline ILP

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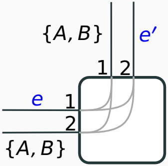


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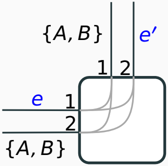


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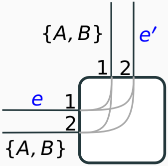


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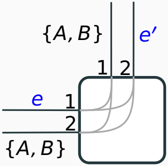


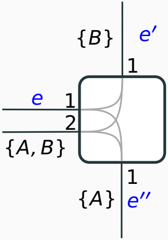


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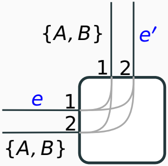


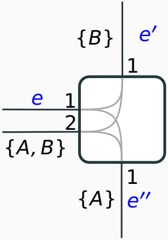


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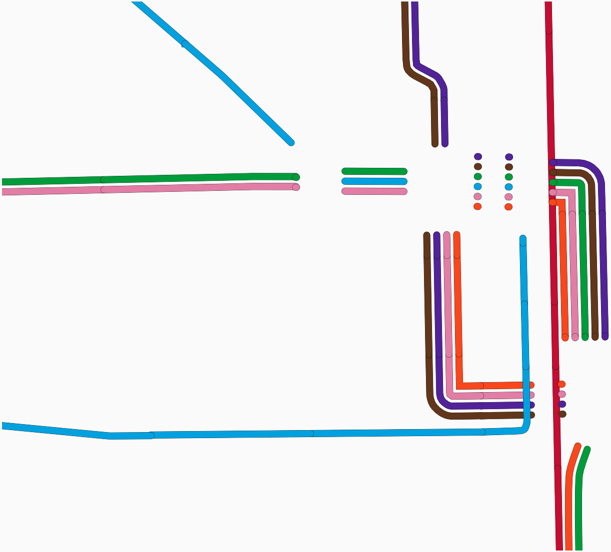

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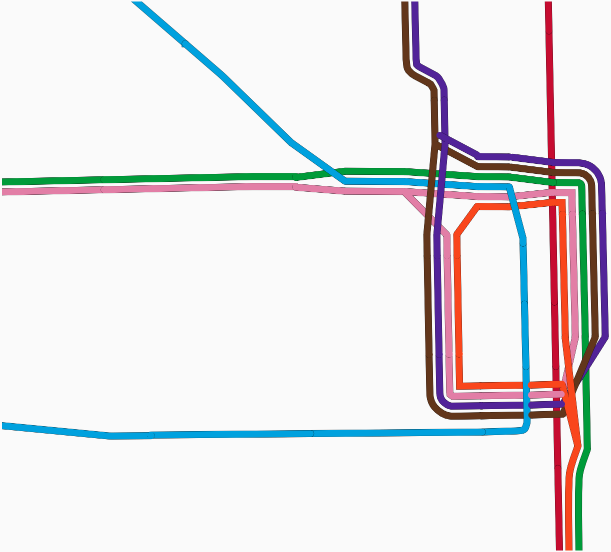

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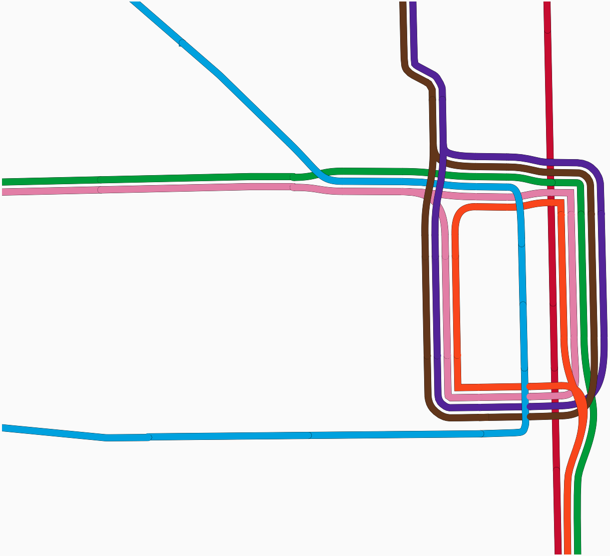

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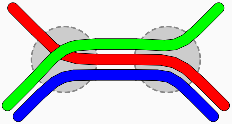

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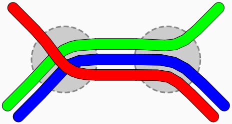

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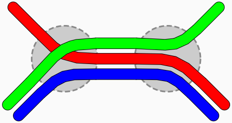

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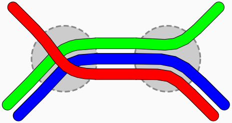

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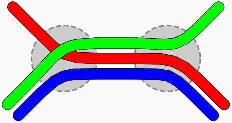

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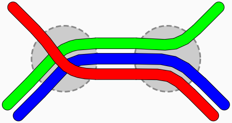

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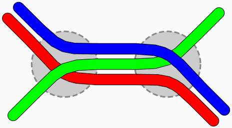

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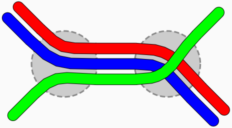

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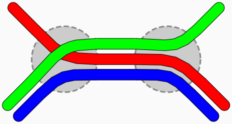

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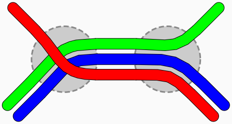

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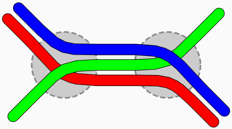

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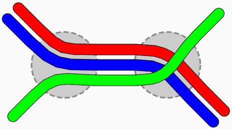

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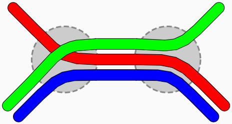

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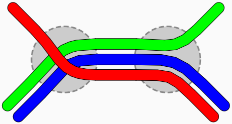

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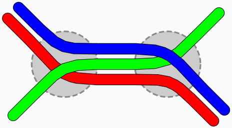

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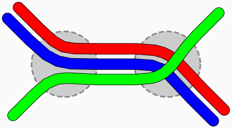

2 crossings

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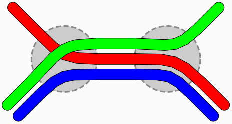
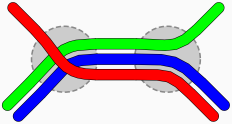
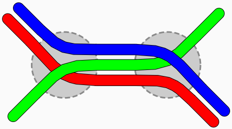


1 crossing
1 separation

vs.



2 crossings
0 separations



2 crossings
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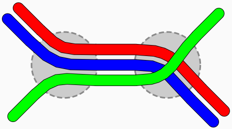
vs.



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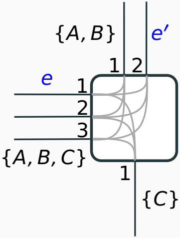
# Line-ordering optimization - Line separations (ctd.)




- **Idea:** If two lines $A$, $B$ continue from $e$ to $e'$, set a binary separation variable $x_{ee'A||B} = 1$ if they are next to each other in $e$, but no in $e'$

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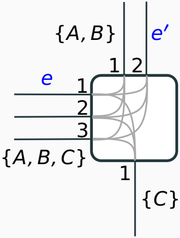


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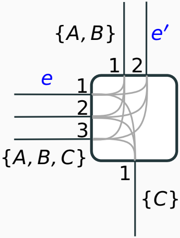


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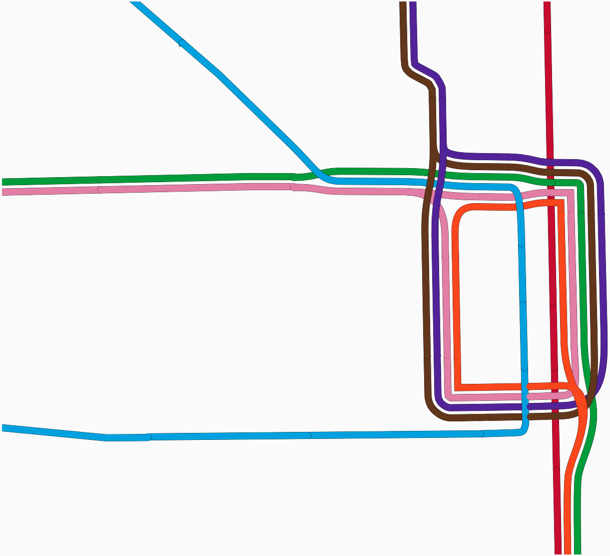
$\Rightarrow$ Still $\mathcal{O}(|E|M^2)$ variables, $\mathcal{O}(|E|M^2)$ constraints

# Results so far (3)



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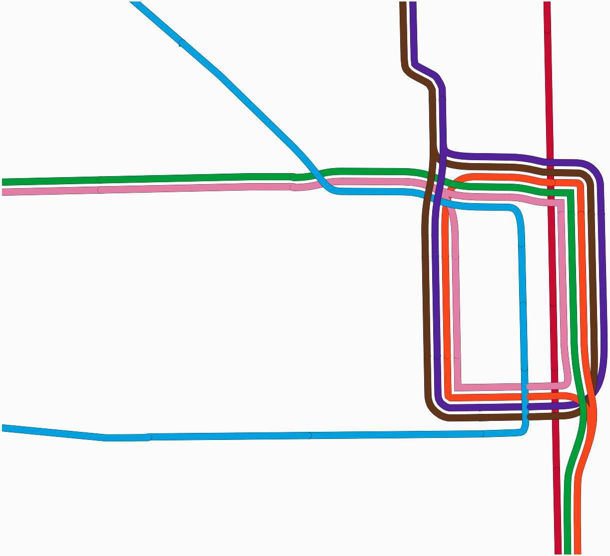
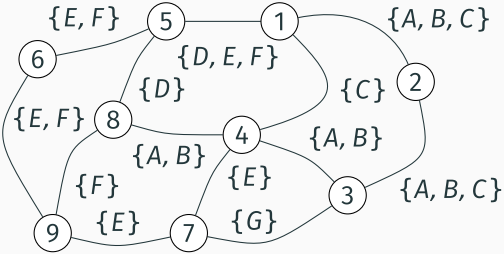


# Line-ordering optimization - Core optimization graph



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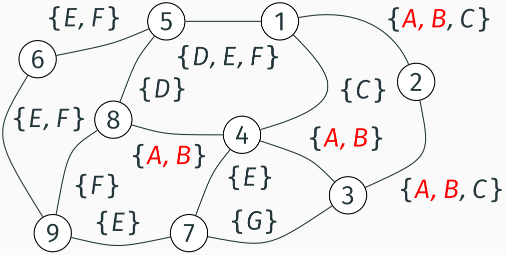

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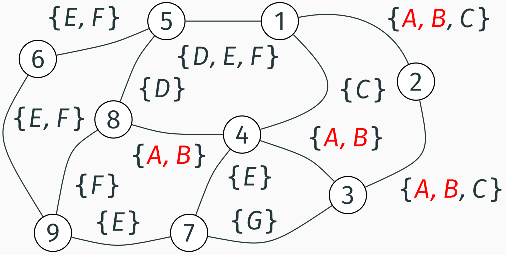


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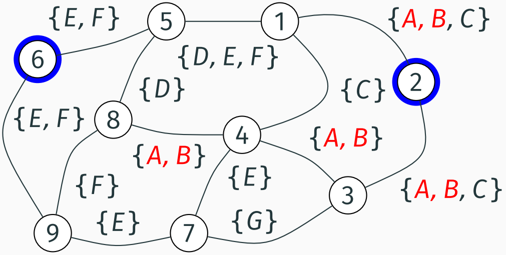


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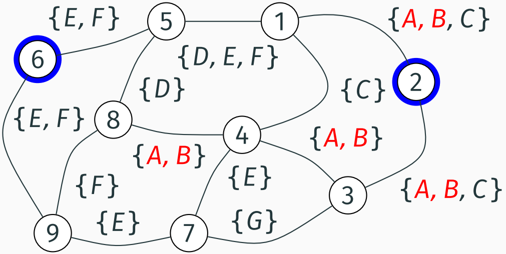


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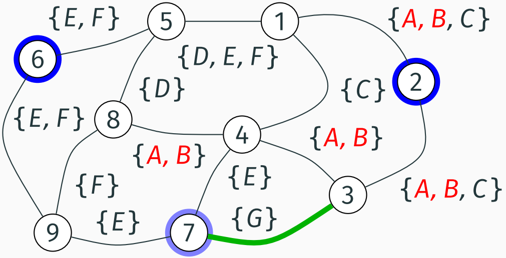


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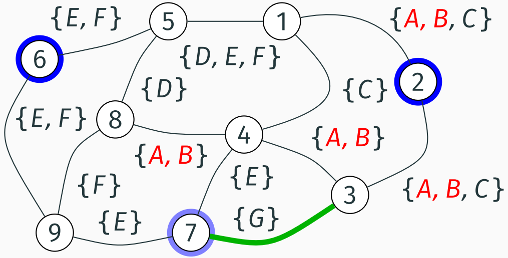


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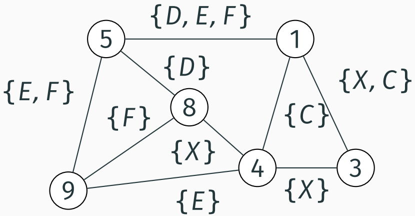


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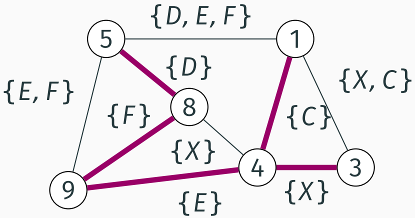


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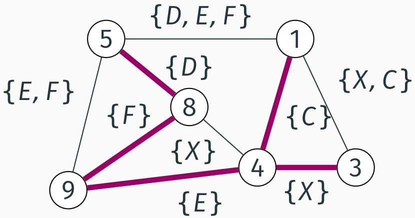


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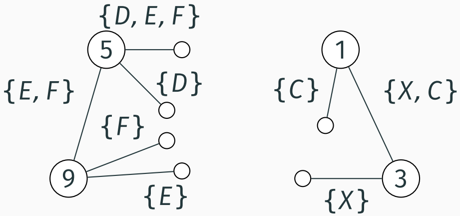


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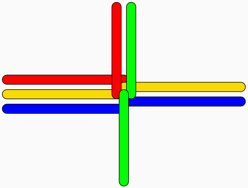

1. Render parallel lines

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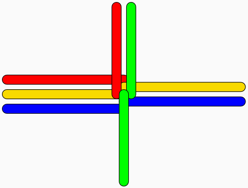

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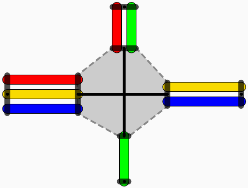
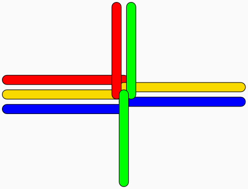


2. Free node space

# Rendering



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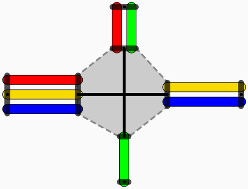

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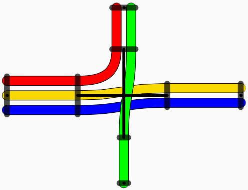

3. Render inner node connections

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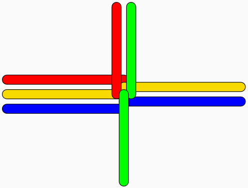

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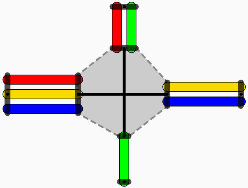

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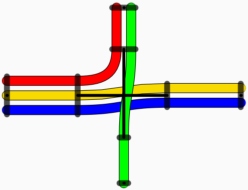

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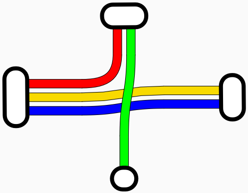
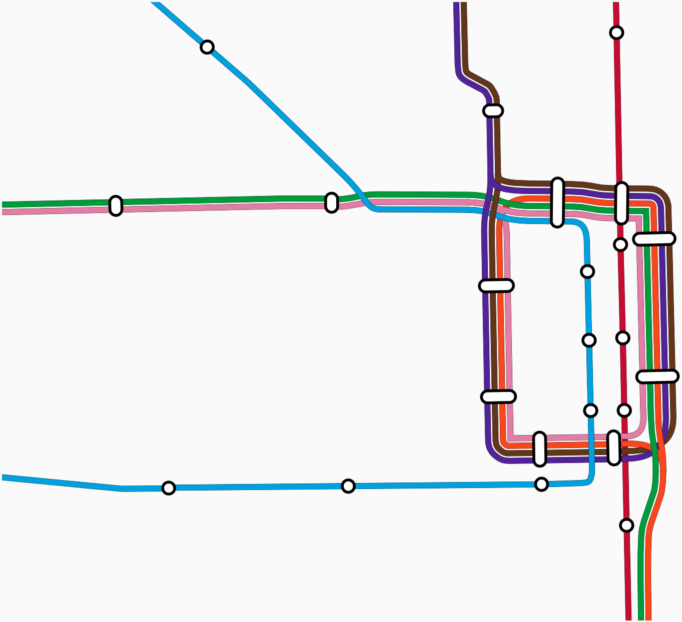


4. Render stations

# Results so far (4)



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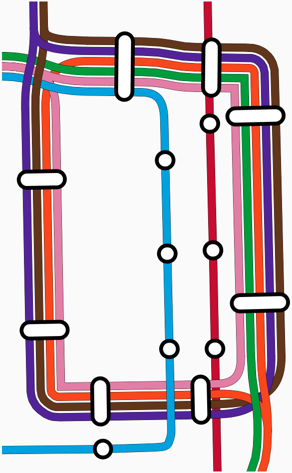

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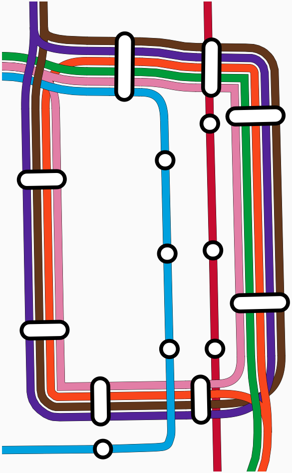

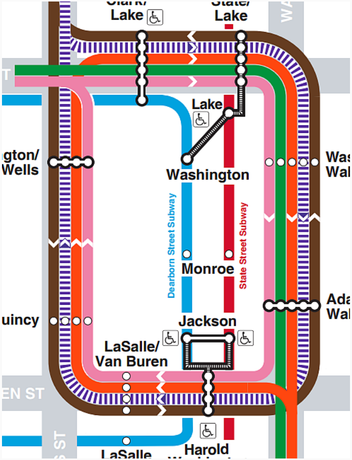

# Evaluation - ILP Solution times

ILP solution times for Chicago, on baseline graph

| | rows × cols | GLPK | CBC | GU | × | \|\| |
|---|---|---|---|---|---|---|
| Base | 41k×861 | — | — | — | 22 | 4-7 |
| Impr. | 1.4k×982 | **9s** | **1s** | **41ms** | 22 | 4-7 |
| + Sep. | 1.9k×1.2k | **47m** | **19s** | **1.8s** | 27 | 0 |

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ILP solution times for Chicago, on core graph

| | rows × cols | GLPK | CBC | GU | × | \|\| |
|---|---|---|---|---|---|---|
| Base | 8.2k×266 | — | **47m** | **2m** | 22 | 4-7 |
| Impr. | 394×285 | **0.8s** | **0.1s** | **10ms** | 22 | 4-7 |
| + Sep. | 505×338 | **23s** | **3.8s** | **0.3s** | 27 | 0 |

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- Additional rules for core graph reduction
  (work in progress)

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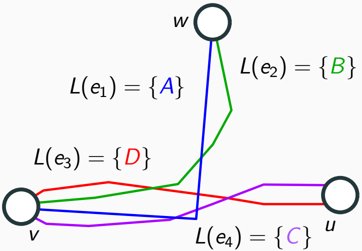
- Octilinearize line graph for (non-overlay) schematic metro maps (work in progress)

# Thank you!

http://loom.informatik.uni-freiburg.de

# Line graph construction - Shared segment collapsing



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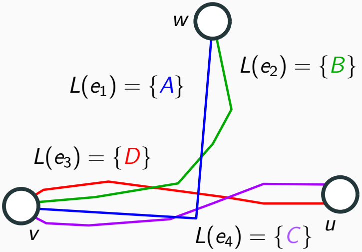


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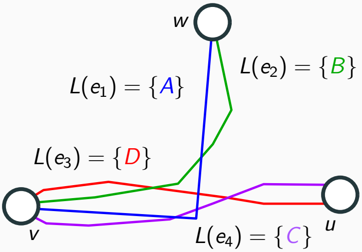


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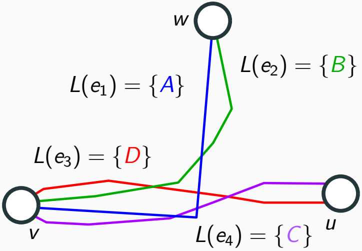


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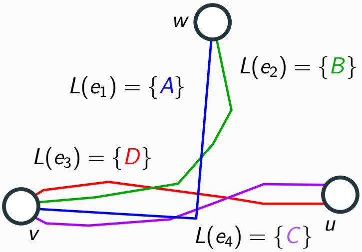


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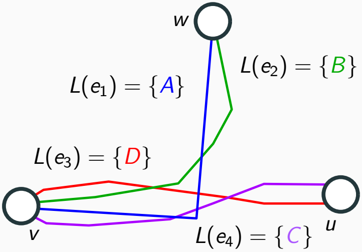


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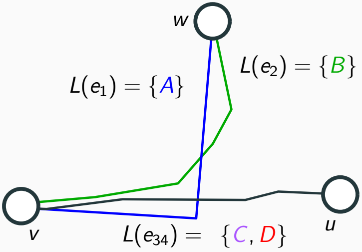


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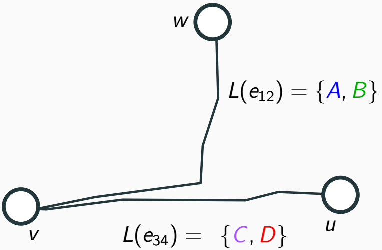


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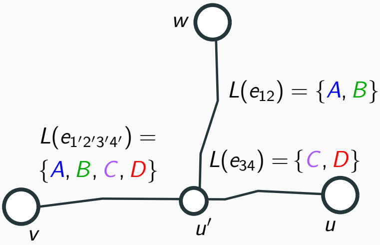


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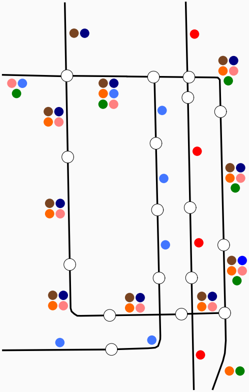
# Evaluation - Line Ordering

T = number of (consecutive) line swaps necessary to transform offical map into our map

| | Off. map | | Our map | | |
|---|---|---|---|---|---|
| | × | \|\| | × | \|\| | **T** |
| Freiburg | 7 | 1 | 7 | 0 | **2** |
| Dallas | 3 | 1 | 3 | 0 | **1** |
| Chicago | 26 | 0 | 27 | 0 | **1** |
| Stuttgart | 65 | 5 | 64 | 2 | **4** |

# Line-ordering optimization - Exhaustive approach



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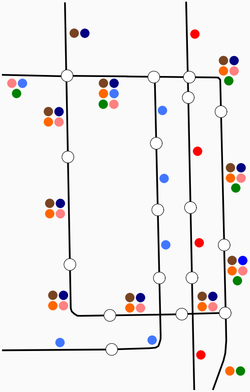

- 23 edges

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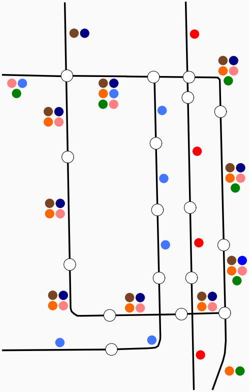

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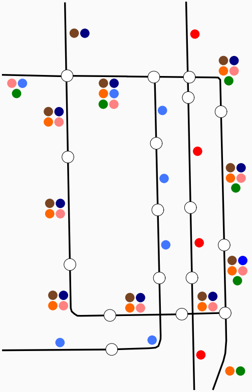

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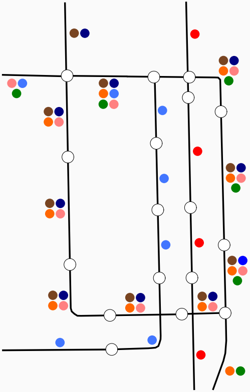

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$\Rightarrow$ **Naive exhaustive search infeasible**

# Baseline ILP - Details

Each line must only be assigned one position:

$$\forall l \in L(e) : \sum_{p=1}^{|L(e)|} x_{elp} = 1.$$

Each position must only be assigned once:

$$\forall p \in \{1, \ldots, |L(e)|\} : \sum_{l \in L(e)} x_{elp} = 1.$$

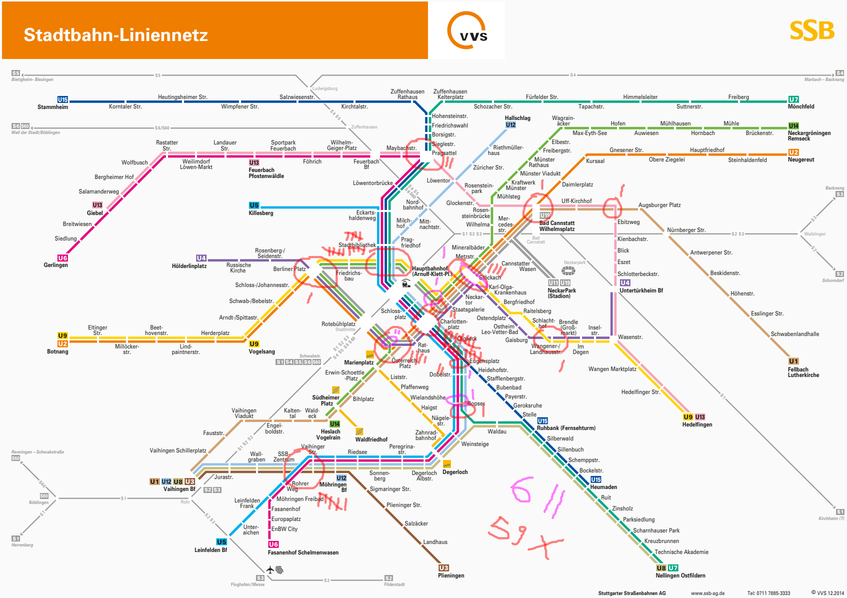
Constraints for ensuring that $x_{ee'AB} = 1$ if a crossing occurs:

$$x_{eA1} + x_{eB2} + x_{e'A2} + x_{e'B1} - x_{ee'AB} \leq 3$$

$$x_{eA2} + x_{eB1} + x_{e'A1} + x_{e'B2} - x_{ee'AB} \leq 3$$

...etc

# Stuttgart map - annotated



# Dataset dimensions

| | $t_{\text{extr}}$ | $\lvert\mathcal{S}\rvert$ | $\lvert V\rvert$ | $\lvert E\rvert$ | $\lvert\mathcal{L}\rvert$ | $M$ |
|---|---|---|---|---|---|---|
| Freiburg | 0.7s | 74 | 80 | 81 | 5 | 4 |
| Dallas | 3s | 108 | 117 | 118 | 7 | 4 |
| Chicago | 13.5s | 143 | 153 | 154 | 8 | 6 |
| Stuttgart | 7.7s | 192 | 219 | 229 | 15 | 8 |
| Turin | 4.9s | 339 | 398 | 435 | 14 | 5 |
| New York | 3.7s | 456 | 517 | 548 | 26 | 9 |

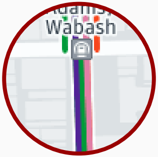
# Core graph dimensions

| | $\|V\|$ | $\|E\|$ | $\|\mathcal{L}\|$ | $M$ |
|---|---|---|---|---|
| Freiburg | 20 | 21 | 5 | 4 |
| Dallas | 24 | 24 | 7 | 4 |
| Chicago | 23 | 24 | 8 | 6 |
| Stuttgart | 50 | 58 | 15 | 8 |
| Turin | 91 | 124 | 14 | 5 |
| New York | 110 | 138 | 23 | 9 |

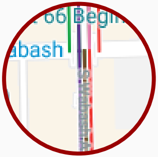
# Challenges - Detail

Official

HERE

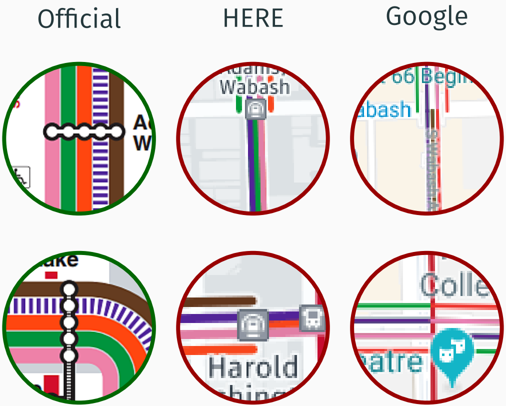



Google



I. Avoid line overlaps

# Challenges - Detail

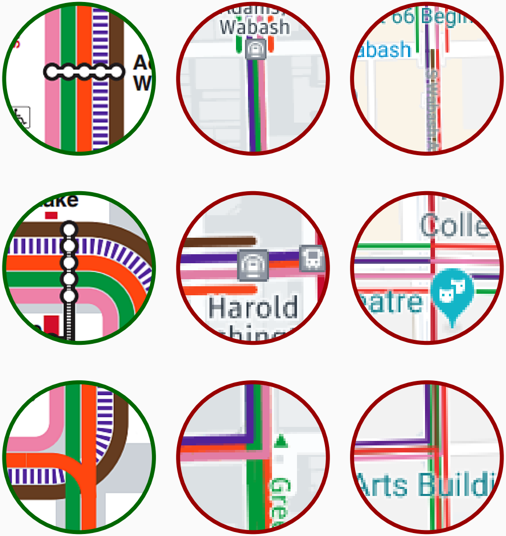



I. Avoid line overlaps

II. Match line orderings

# Challenges - Detail

Official HERE Google




I. Avoid line overlaps

II. Match line orderings

III. Clearly indicate line continuations