The Hierarchy in Grid Graphs

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How to efficiently find shortest paths in grid graphs using Contraction Hierarchies (CH)

Contraction Hierarchies (CH) [Geisberger et al. 2008]: Dijkstra-based speed-up technique to accelerate shortest path queries in road networks $G(V, E, c)$

Basic Construction:
1. enumerate all nodes due to some notion of importance $L : V \rightarrow N$
2. contract nodes one-by-one wrt $L$, maintain shortest paths between remaining nodes by inserting additional edges (=shortcuts)
3. build new graph $G'(V, E', c')$ with $E' = E \cup$ shortcuts

In $G'$ an edge $e = (v, w)$ is called upward if $L(v) < L(w)$, downward otherwise. A path is called upward/downward, if it consists of upward/downward edges only.

Maintaining Canonical Paths

Goal: find shortest path with minimal number of turns
- saves energy
- more natural way of moving

Approach:
1. assign edge classifiers to the shortcuts, indicating the type (v-vertical or h-horizontal) of the first and the last edge of the spanned path as well as the number of turns ($t$)
2. maintain in the CH-construction all shortcuts which are part of canonical paths
3. adapt the CH-search algorithm to the classifiers such that canonical paths can be found in the CH-graph

Theorem: For every pair of vertices $s$, $t \in V$ let $G^u(s)$ be the set of all upward paths starting in $s$ and $G^d(t)$ all downward paths ending in $t$, then a bidirectional Dijkstra in $G^u(s) \cup G^d(t)$ provides the optimal path distance.

CH-GRAPH

For given source/target $s$, $t \in V$ let $G^u(s)$ be the set of all upward paths starting in $s$ and $G^d(t)$ all downward paths ending in $t$, then a bidirectional Dijkstra in $G^u(s) \cup G^d(t)$ provides the optimal path distance.

Experimental Results

For mazes and rooms CH-search expands less nodes than the optimal path size, i.e. any search algorithm which is not based on path compression can not perform better than our approach!

<table>
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<th>input type</th>
<th>avg. path size</th>
<th># of expanded nodes</th>
<th>A*</th>
<th>CH-search</th>
<th>CH-A* uni</th>
<th>speed-up</th>
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